2015 SECME Regional
Mathematics Contest
High School

Exam, Form: $\boxed{\text{A}}$

Date: February 14, 2015

Answer the questions in the space provided below. Show your work on the scrap paper you have been provided. Make sure you clearly label the work for each problem. Include units when necessary.

1. Find $x^{\frac{-y}{2}}$ given the following system:

$$x + 3y = 19$$
$$\frac{1}{2}x - 2y = -8.$$

2. Find x if $3^{x^2+x+1} + 3^{x^2+x+1} + 3^{x^2+x+1} = 3^{44}$.

3. What is three multiplied by the sum of the integers from 3 to 57?

4. Consider two idential intersecting circles such that A is the center of the left circle and C is the center of the right circle. Given that triangle ABC is an equilaterial triangle with perimeter 15, find the perimeter of the shaded region.
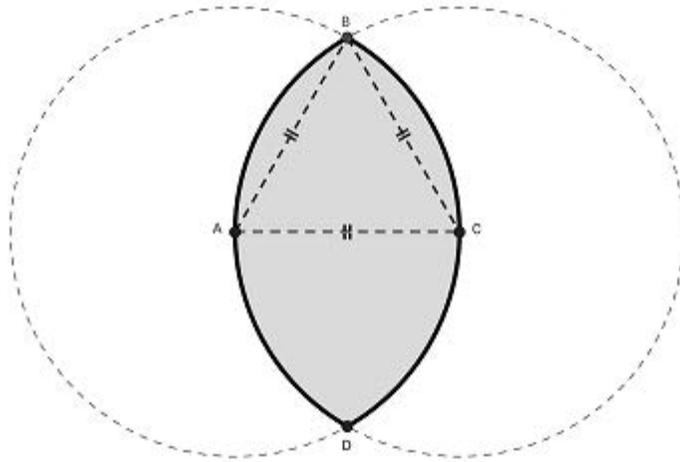
5. Magic Square: Fill in the figure with the numbers 1 through 9 such that each number is used exactly once and the sum of each row, column, and main diagonal is equal to 15.
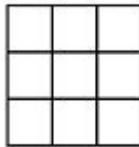
6. Factor the following polynomial
$$f(x) = 2x^3 + 11x^2 - 7x - 6$$

7. What is $g(f(g(f(x))))$ is $f(x) = 2\sqrt{x}$ and $g(x) = x^2$.

8. Given that $\csc(\theta) = \dfrac{5}{4}$, find $\tan(\theta)$.

9. Write the following as a single logarithm

$$f(x) = \ln(x^2 - 1) - 3\ln(x + 1).$$

Then how many real zeros does this function have?

   (a)   One real zero

   (b)   Two real zeros

   (c)   Infinite real and imaginary zeros

   (d)   No real zeros

   (e)   Not enough information

10. The students at a high school were surveyed regarding their high school lunch. 12% said it was good, 54% said it was bad, and the rest of the students said no opinion. If 650 students were surveyed, how many of the students said no opinion.

11. Find θ where the following parallelogram has perimeter 44 cm and area 64 cm^2.
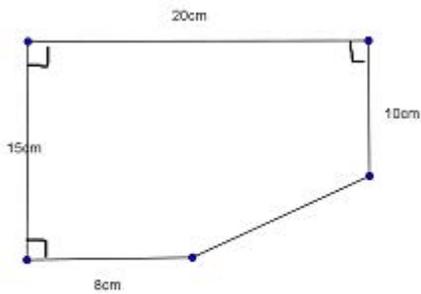
3x+2

Θ

5x+4
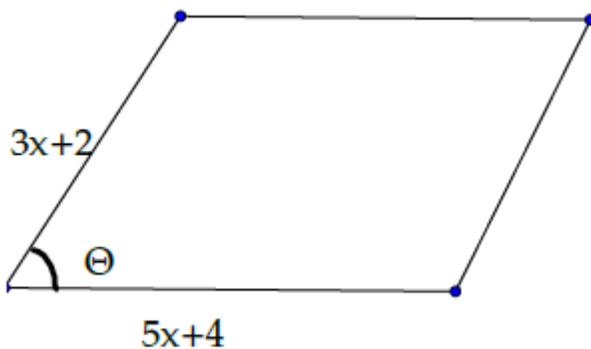
12. You are designing a rectangular room and you know that you need the perimeter to be 400 m. What are the dimensions of the rectangular room so that the area is the largest?
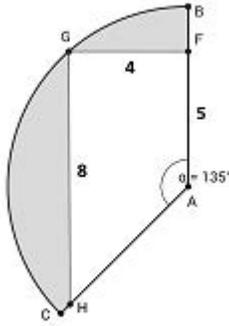
13. Solve the triangle:
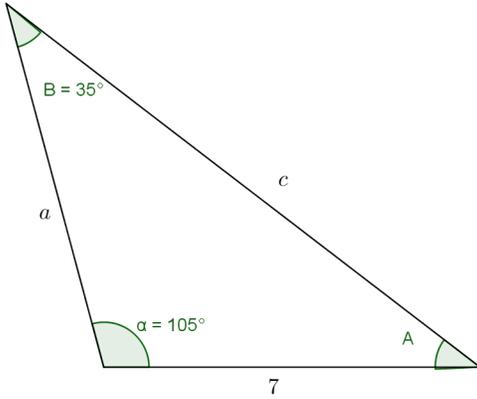
14. Find the area of the region that is bounded by $y \leq x + 3$, $y \leq -x + 3$, and $y \geq 0$.
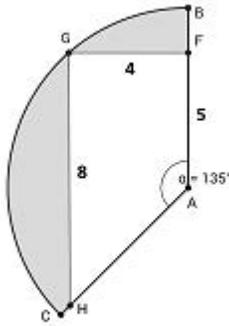
15. Find the area for the given shape:

16. Find the area of the shaded region given that the radius AB is length 6.

17. Solve for x in $e^{2x} - 9e^x + 18 = 0$.

18. Find x, y, and z if

$$x + y - z = 6$$
$$3x - 2y + z = -5$$
$$x + 3y - 2x = 14.$$

Answer Key for Exam $\boxed{\text{A}}$

1. $x^{-\frac{y}{2}} = \dfrac{1}{32}$

2. $x = 6$ or $x = -7$ (both answers are required)

3. 4960

4. $\dfrac{20\pi}{3}$

5. There are multiple correct answers. Check that the sum of each add to 15.

6. $f(x) = 2(x+6)\left(x + \dfrac{1}{2}\right)(x-1)$ or $f(x) = (x+6)(2x+1)(x-1)$

7. $16x$

8. $\tan(\theta) = \dfrac{4}{3}$

9. $f(x) = \ln\left(\dfrac{x-1}{(x+1)^2}\right)$
   (d) No real zeros

10. 221 students

11. $\theta = \sin^{-1}\left(\dfrac{4}{7}\right)$.

12. 100 m $\times$ 100 m

13. $A = 40°$
    $a = \dfrac{7\sin(40°)}{\sin(35°)}$
    $c = \dfrac{7\sin(105°)}{\sin(35°)}$.

14. 9

15. 270 cm^2

16. $\dfrac{27\pi}{2} - 26$

17. $x = \ln(6), \ln(3)$ (both answers are required)

18. $x = 1,\ y = 3,\ z = -2$
    $(1, 3, -2)$ is also an acceptable answer